Linear Differential Systems with Infinite Power Series Coefficients

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Abstract

We consider the following problem: given a linear ordinary differential system S of arbitrary order with infinite formal power series coefficients, decide whether the system has non-zero Laurent series, regular, or formal exponential-logarithmic solutions, and find all such solutions if they exist. If the coefficients of the original systems are arbitrary formal power series represented algorithmically (thus we are not able, in general, to recognize whether a given series is equal to zero or not) then these three problems are algorithmically undecidable. But, it turns out that the first two problems are decidable in the case when we know in advance that a given system is of full rank. However, the third problem (finding formal exponential-logarithmic solutions) is not decidable even in this case. We specify a limited version of the third problem, for which there is a required algorithm: namely, if S and a positive integer d are such that for the system S the existence of d linearly independent solutions is guaranteed, we can build these d solutions.

It is shown also that the algorithmic problems connected with the ramification indices of irregular formal solutions of a given system are mostly undecidable even if we fix a conjectural value r of the ramification index. This enables us to obtain a strengthening of the theorem that we are not able to compute algorithmically the dimension of the space of all formal solutions although we can construct a basis for the subspace of regular solutions. In fact, it is impossible to compute algorithmically this dimension even if, in addition to the system, we know the list of all values of the ramification indices. However, there is nearby an algorithmically decidable problem: if a system S and positive integers r, d are such that for S the existence of d linearly independent formal solutions of ramification index r is guaranteed then one can compute such d solutions of S.

We prove additionally that the width of a given full rank system S with formal power series coefficients can be found algorithmically, where the width of S is the smallest non-negative integer w such that any l-truncation of S with $l \geqslant w$ is a full rank system. An example of a full rank system S and a non-negative integer l such that l-truncation of S is of full rank while its $(l+1)$-truncation is not, is given; however it is shown as well that the above-mentioned value w exists for any full rank system.

Thus, a neighborhood of algorithmically solvable and unsolvable problems is observed.

For the solvable problems mentioned above, we propose corresponding algorithms and their Maple implementation, and report some experiments.